Referee's report on "Online Competitive Algorithms for Maximizing Weighted Throughput of Unit Jobs" by F. Chin, M. Chrobak, S. Fung, W. Jawor, J. Sgall, and T. Tichý

The paper by Chin et al. considers the online buffer management problem that arises in networks supporting Quality-of-Service (QoS) applications. Packets with different QoS values arrive at a network switch and are to be sent along an outgoing link. Due to overloading conditions, some packets have to be dropped. The objective is to maximize the total value of packets that are sent. The authors formulate this as an online unit-job scheduling problem where each job is specified by its release time, deadline, and a non-negative weight representing its QoS value. The goal is to maximize the weighted throughput, that is the total weight of scheduling jobs.

The authors present several competitive online algorithms for various versions of unit-job scheduling, as well as some lower bounds on the competitive ratios. In several cases their upper and lower bounds are tight. A good summary of their results is given in the abstract.

The paper is very well written and technically deep. The results obtained are correct and interesting and the topic of the paper is relevant to the Journal on Discrete Algorithms. Therefore, I recommend to accept the paper for publication in the journal. Since I only found some minor issues, I do not need to see a revision of it.

Below is a list of detailed comments.

1. Page 7, case $j \in Y \cap X$: You do an implicit case distinction here that took me a while to catch and therefore made it hard for me to understand the proof at the beginning. You should explicitly distinguish here between the case that x is selected so that $x \leq \ln(w_j/w_h)$ and the case that x is selected so that $x > \ln(w_j/w_h)$. In the first case, w_j is considered by the online algorithm, and therefore $w_f - \Delta\Phi = w_f$ because Φ does not change, as you argue correctly. In the second case, w_j is not considered by the online algorithm but you can at least guarantee that $w_f - \Delta\Phi \geq 0$. Making this case distinction more explicit makes the proof much easier to understand.

2. Page 7, line 15: "we we" $\rightarrow$ "we"

3. Page 8, line 21, "$\Phi_{x\sigma x} = \ldots$": The second x should be a z. You should better write here "If $z = 0$ then $\Phi_{x\sigma z} = 0$ and otherwise $\Phi_{x\sigma z} = \ldots$".

4. Page 9, paragraph after the proof of Theorem 4.1: You already mentioned this in the proof of Theorem 4.1. No need to repeat this here again.

5. Page 10, proof of Theorem 5.2: This proof is quite long. It would make it easier to understand it if it was better structured. First of all, the proof consists of a part showing that the competitive ratio of $\text{EDF}_{1/\lambda}$ is at most λ and a part (starting on page 13) showing that this competitive ratio is tight. The first part consists of several subparts:

   - A proof that one can reduce every instance to an instance in which all jobs have weights λ^i.
   - A proof showing a competitiveness of λ for the case that f receives a single charge or $w_f = M_t$.
   - A proof for the case that f receives two charges and $w_f = M_t/\lambda$.
   - A proof that $\lambda \geq 2 - (1 - 1/\lambda)/(W + 1)$ in any case in which the integers x_i satisfy (8). (This seems to be an optimization problem that can be solved independently of the scheduling problem that motivated it.)

You may think about formulating lemmata or claims for these subparts to make the proof easier to digest.

6. Page 11, line 3 of case (I): “If j scheduled” $\rightarrow$ “If j is scheduled”

7. Page 11, line 5 of case (I): “If j not scheduled” $\rightarrow$ “If j is not scheduled”

8. Page 11, line -8: “let X_i, be the” $\rightarrow$ “let X_i be the”

9. Page 12, line 5: “scheduled by in E” $\rightarrow$ “scheduled in E”

10. Page 12, line 8: Rephrase “Since the span of job j covers all of this and at least one time step after A finishes $X_{i'}$, we have...”

11. Page 12, line 13: “Using (III)” $\rightarrow$ “Using (II)”?

12. Page 12, line 14: “total at most” $\rightarrow$ “a total of at most”

13. Page 12, line 17: “increases with W, thus we need” $\rightarrow$ “increases with W. Thus, we need”

14. Page 13, first paragraph: Replace “EDF” by “EDF$_{1/\lambda}$”. For completeness, you should give expressions for the total weight scheduled by your online algorithm and the total weight scheduled by the optimal algorithm.